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Optical Glass Compatibility For the Design of Apochromatic Systems

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Abstract:

The design of apochromatic systems is difficult because of two problems: the glass sorts compatibility and the c_1/c_a arbitrary input ratio. The optical glass manufacturers offer a wide range of sorts, so that the choice of triplet compatible glasses becomes itself an important separate problem. The paper provides a solution of mathematical modeling for the glass compatibility and, practically, analyses the sorts presented by Schott GmbH. The original software provided 22 compatible glass triplets. The authors explored the possibilities of enlarging the c_1/c_a ratio from the value 0.6 indicated in the literature to a range of [0.5…0.8]. Therefore, they designed and analyzed a set of 88 triplets. A correct glass choice can insure twice-larger apertures than the traditional ones for best quality apochromats (diffraction-limited).

Keywords: *Optical glass compatibility, Triplet design, Image quality, Large aperture, Aspherical surface*

1. Introduction

Apochromatic optical systems need a specific combination of optical glass sorts, whose dispersive properties insure correction of the longitudinal chromatic aberration and secondary spectrum. An apochromatic system accomplishes the superposition of image abscissas for three wavelengths, so that the secondary spectrum is much lower than for any other optical entity. The traditional optical system, which satisfies these conditions, is the cemented apochromatic triplet (fig.1). The design algorithm supposes that the operator has already chosen the glass sorts. Existing literature offers only general recommendations or a minimum number of compatible glasses [1, 2]. Efficient use of a large number of glass sorts needs a mathematical approach.

The database to investigate contains the glass sorts offered by the Schott Catalogue. The parameters taken into account are the refractive indexes of the spectral lines g, F', e, C', s, the Abbe number ν_e and the relative partial dispersion P_{ge}. The Schott Catalogue contains

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about 200 glass sorts. The choice of three compatible glasses from a 200 elements string is difficult or impossible to achieve by permutations or randomization, not to mention the time needed. The designer should find some mathematical criteria, which can lead to exact solutions.

2. Theoretical basis

The design of the cemented triplet intends to correct the longitudinal chromatic aberration and the secondary spectrum using the following system of equations [1]:

$$\begin{cases} c_a = \dfrac{1}{f'E(\nu_a - \nu_c)}\left(\dfrac{P_b - P_c}{\Delta n_a}\right) \\ c_b = \dfrac{1}{f'E(\nu_a - \nu_c)}\left(\dfrac{P_c - P_a}{\Delta n_b}\right), \\ c_c = \dfrac{1}{f'E(\nu_a - \nu_c)}\left(\dfrac{P_a - P_b}{\Delta n_c}\right) \end{cases} \tag{1}$$

where c is the curvature of the elements, f' – the effective focal length of the triplet, ν - the Abbe number (ν_e) , Δn – the main dispersion, P – the relative partial dispersion, a, b, c – the indexes of the three component lenses.

In the expression of the curvatures (1), the denominator contains the size E:

$$E = \frac{\nu_a(P_b - P_c) + \nu_b(P_c - P_a) + \nu_c(P_a - P_b)}{\nu_a - \nu_c}, \tag{2}$$

which depends on the dispersive parameters of the glasses. The previous design experience shows that proper glasses to form a triplet must cover characteristics satisfying the relations:

$$\nu_a > \nu_b > \nu_c \wedge P_a < P_b < P_c, \tag{3}$$

which indicates the sorts choice in the families FK (Flour - Crown) – KzFS (special short Flint) – SF (dense Flint), [1], [2].

The size E has no physical significance and it is not a dispersive parameter, however, it is very important. From the algebraic point of view, E must be a negative number and an absolute value, E should be as large as possible. Respecting these conditions, positive curvatures for the lenses a and c and negative curvature for the lens b are obtained. Also, small curvatures (respectively, large radii and small powers) are insured.

Usually, large amounts of geometrical aberrations, especially spherical, which are hard or impossible to correct, affect the resulting systems. This occurs because the design of the triplet only considers correction of chromatic aberrations and the secondary spectrum. Therefore, the triplets are always low speed systems, limited to f-numbers in the range from f/8 to f/10, due to uncorrected geometrical aberrations.

Since the glass sort offer became larger and larger, original selection software was developed. The appropriate combinations for the apochromat triplet were found using the Schott database, [3], the selection conditions (3), completed with the condition ABS(E)>0.0145.

2. Numerical results

Appropriate combinations for the cemented apochromatic triplet, obtained by using the original software, are presented in Tab. I.

Tab. I Twenty-two solutions of compatible glasses for apochromatic triplets

Glass combination ID	E	Sort a	Sort b	Sort c
	1	2	3	4
1	-0.01455	FK51	KZFS1	TIF6
2	-0.01517	FK51	KZFSN2	TIF6
3	-0.01501	FK52	KZFSN2	TIF6
4	-0.01494	FK54	BK3	TIF6
5	-0.01517	FK54	KZFS1	TIF6
6	-0.01485	FK54	KZFS6	TIF6
7	-0.01547	FK54	KZFSN2	SF57
8	-0.01486	FK54	KZFSN2	SF58
9	-0.01546	FK54	KZFSN2	SF59
10	-0.01623	FK54	KZFSN2	TIF6
11	-0.01470	FK54	KZFSN9	TIF6
12	-0.01470	FK54	LAK16A	TIF6
13	-0.01461	FK54	LAK28	TIF6
14	-0.01501	FK54	LAK31	TIF6
15	-0.01466	FK54	LAK33	TIF6
16	-0.01492	FK54	LAK8	TIF6
17	-0.01463	FK54	LAK9	TIF6
18	-0.01503	FK54	LAKL21	TIF6
19	-0.01498	FK54	LAKN14	TIF6
20	-0.01492	KZFS7A	SFL57	TIF6
21	-0.01495	KZFSN5	SF58	TIF6
22	-0.01541	KZFSN5	SF59	TIF6

The first column indicates the value of the size E, computed with relative dispersions $P_{e\text{-}g}$ and Abbe numbers ν_e. The columns 2-4 indicate suitab. sorts for apochromatic triplets with the structure: positive-negative-positive elements.

3. Discussion on applications

Based on the results in tab. I, a study was developed, which included synthesis and the analysis of 88 triplets. The design input data was f'=100, s= -∞ (object abscissa), $\omega=5^o$ (half – field). For each glasses combination, four options were taken into account: $0.5c_a$, 0.6 c_a, 0.7 c_a and 0.8 c_a, representing values of the first curvature. Regarding this aspect, the literature indicates a c_1/c_a ratio of about 0.6, [1, 2]. The design algorithm was enriched by widening the ratio range and, practically, all possible solutions were found.

The triplets designed using original software were analyzed with the program OSLO LT, from Lambda Research Ltd.

The study leads, synthetically, to the following remarks:

- ❒ The glass choice and the c_1/c_a ratio influence the shape of the lenses
- ❒ There are seven possible combinations of lens shapes, presented in fig. 2
- ❒ The most significant influence parameter on the image quality is the materials combination (the relationship between the refractive and dispersive characteristics)
- ❒ The best solutions follow the C(convergent)- D(divergent) – C(convergent) scheme and also, global biconvex or convergent meniscus shape

❒ From the diffraction point of view, the best quality is obtained for the c_1/c_a ratio within the range (0.5…0.6)
❒ The most favorable glass combinations for the first lens contain a sort from the flour-crown family (FK), for the middle lens a sort from the short flint (KZFS) or crown-lanthanum (LAK) and for the last lens a sort from flint-lanthanum family (TIF)
❒ Diffraction limited systems result only for f-numbers smaller than f/5. The primary solutions, obtained by applying the design algorithm, are obviously not high quality systems, therefore they can be improved by defocusing
❒ Bending is not at all efficient in increasing quality, thus the geometry found by applying the algorithm is unchangeable
❒ The triplets, which were studied, are very good from the chromatic point of view, meaning that the longitudinal chromatic aberration and the secondary spectrum are corrected, but the spherical residual aberration, which establishes the aperture, cannot be decreased.

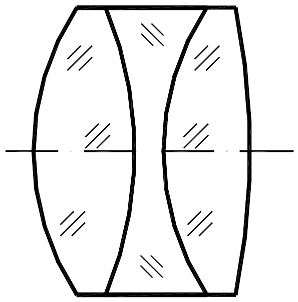


Fig. 1 Apochromatic cemented triplet

The apochromats obtained with two glass combinations are briefly presented in tab. II. The diffraction-limited solutions are indicated with grey filling. The yellow filling marks the largest aperture of the solution preserving the diffraction-limited character.

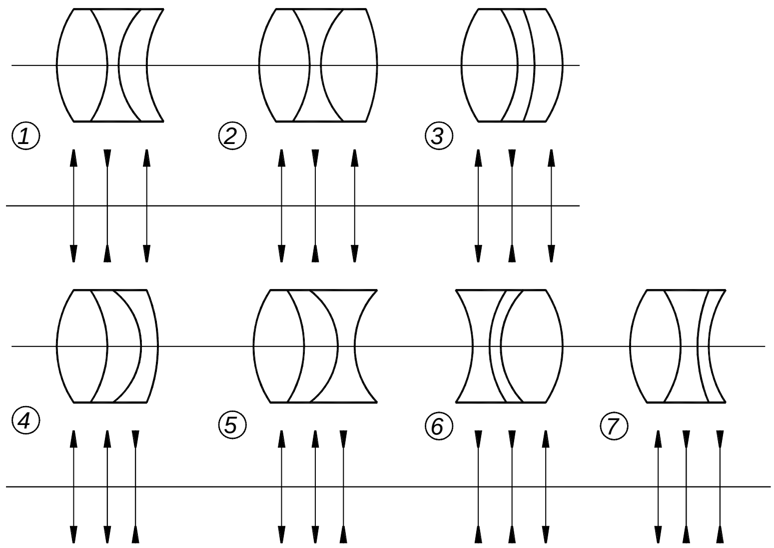
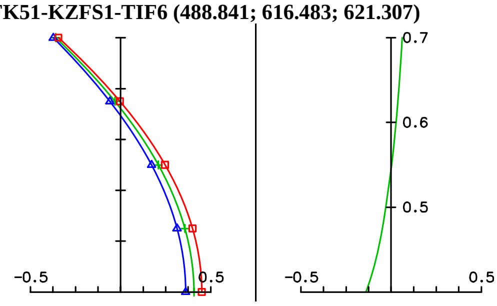
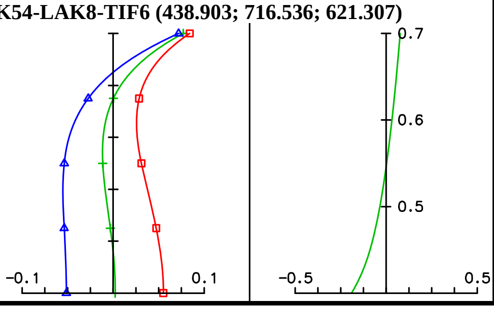


Fig. 2 Seven possible shapes of lenses forming an apochromatic triplet

The glass combinations with codes attached and a brief graphical description of aberrations (spherochromatism and chromatic focal shift) are presented in column 1. According to the numbers in fig. 2, the shape of the triplets is indicated in column 2 for each glass combination. The c_1/c_a ratio is presented in column 3. The f-number, respectively the incidence height are shown in the next two columns. The last two columns, 6 and 7, provide the quality parameters RMS OPD and Strehl ratio.

Tab. II. Diffraction limited apochromats designed using the mathematically chosen compatible glass triplets

f'=100, s= -∞, ω=5°							
ID	**sorts**	**shape**	**c_1/c_a**	**f'/D**	**h [mm]**	**RMS OPD**	**Strehl**
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1	**FK51-KZFS1-TIF6 (488.841; 616.483; 621.307)** 	2	0.5	1/10	5	0.016	0.963
				1/8	6.25	0.025	0.948
				1/7	7	0.033	0.929
				1/6.25	8	0.049	0.874
				1/5	10	0.124	0.567
				1/4	12	0.318	0.124
		1	0.6	1/10	5	0.036	0.948
				1/8	6.25	0.077	0.785
				1/7	7	0.116	0.527
				1/6.25	8	0.193	0.222
		1	0.7	1/10	5	0.086	0.745
				1/8	6.25	0.195	0.239
		1	0.8	1/10	5	0.154	0.413
				1/8	6.25	0.351	0.114
2	**FK54-LAK8-TIF6 (438.903; 716.536; 621.307)** 	3	0.5	1/10	5	0.097	0.649
		2	0.6	1/10	5	0.033	0.957
				1/8	6.25	0.049	0.911
				1/7	7	0.058	0.879
				1/6.25	8	0.067	0.843
				1/5	10	0.085	0.741
		1	0.7	1/10	5	0.180	0.507
		1	0.8	1/10	5	0.271	0.130

One can notice that there are very good solutions for $c_1/c_a \in \{0.5, 0.6, 0.7, 0.8\}$. The choice for a certain solution can take into account more sophisticated criteria, such as size of radii. Only 53 solutions of 88 are diffraction limited (for f-numbers within the range of f/10…f/5). The diffraction-limited solutions and their acceptab. apertures are synthesized in tab. III.

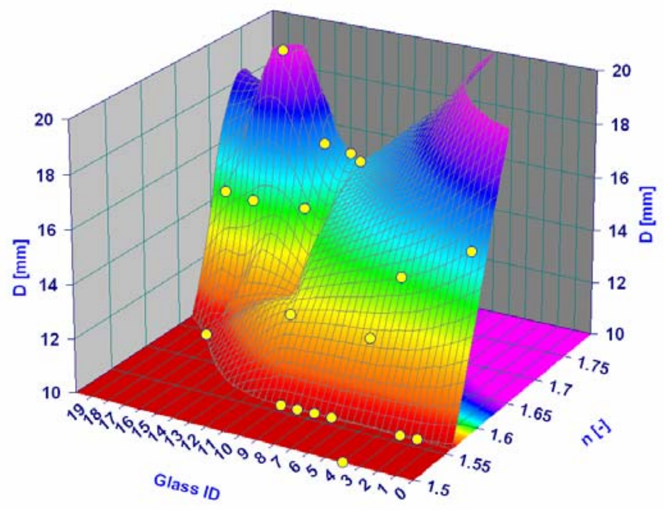


Fig. 3 Course of aperture in relationship with the main refractive index of the middle glass

Tab. III. Synthetic table containing diffraction limited apochromats and their maximum permissible apertures

Glass combination	c_1/c_a	f'/10	f'/8	f'/7	f'/6.25
FK51-KZFS1-TIF6	0.5	x	x	x	x
	0.6	x	x		
FK51-KZFSN2-TIF6	0.6	x			
FK52-KZFSN2-TIF6	0.6	x			
FK54-KZFS1-TIF6	0.5	x	x	x	
	0.6	x	x	x	
FK54-KZFS6-TIF6	0.5	x	x		
	0.6	x			
	0.7	x			
	0.8	x			
FK54-KZFSN2-SF57	0.6	x			
FK54-KZFSN2-SF58	0.6	x			
FK54-KZFSN2-SF59	0.6	x			
FK54-KZFSN2-TIF6	0.6	x			
FK54-KZFSN9-TIF6	0.5	x	x		
FK54-LAK16A-TIF6	0.6	x	x	x	x
FK54-LAK28-TIF6	0.6	x	x	x	x
FK54-LAK31-TIF6	0.6	x	x	x	
FK54-LAK33-TIF6	0.6	x	x	x	x
FK54-LAK8-TIF6	0.6	x	x	x	x
FK54-LAK9-TIF6	0.6	x	x	x	
FK54-LAKL21-TIF6	0.5	x			
	0.6	x			
FK54-LAKN14-TIF6	0.6	x	x	x	

The following discussion refers only to diffraction-limited solutions and represents an analysis to relate the material refractive and dispersive properties and the maximum aperture, which introduces the limitation in apochromats use.

The sorts selected automatically satisfy the relation (3). The effective range of values is presented in tab. IV.

Tab. IV. Refractive and dispersive properties of compatible sorts

Lens	n_e	ν_e	P_{g-e}	ν_{g-e}
a	1.43...1.48	80.07...90.31	0.665...0.668	55.06...61.24
b	1.56...1.75	44.13...64.89	0.663...0.674	29.44...44.08
c	1.62...1.96	20.20...30.66	0.697...0701	12.97...19.77

For most combinations, the first glass is a FK sort and the last one is a TIF6 sort. Therefore, the middle glass gives the specific characteristic of each solution, mainly. Its refractive and dispersive properties strongly influence the spherochromatism and finally decide the reasonable f-number. Tab. V contains the main refractive and dispersive characteristics of the middle sort.

The last columns indicate the largest aperture for which the apochromat is still diffraction-limited, the corresponding f-number and the shape of the system according to the codes in fig. 2.

Column 4 introduces a new parameter, *k*, defined as the ratio ν_e/n_e, which can be relevant in a simple, synthetical manner, for the image quality.

The middle lens is in all cases divergent. The first one is convergent (bi-convex for all shapes – 1, 2 and 7). The third lens is convergent (meniscus for shape 1 and bi-convex for shape 2) or divergent (meniscus for shape 7). The largest apertures are obtained for the shapes 1 and 2. A high refractive index and small Abbe number, hence low values of ratio *k*, facilitate a small residual spherocromatism.

Tab. V. Refractive and dispersive properties of the middle sort

Glass ID	Glass sort	n_e	ν_e	$k=\nu_e/n_e$	D [mm]	f/D	Shape
0	1	2	3	4	5	6	7
1	**KZFS1**	1.6164	29.51	18.25	16.0	1/6.25	2
2	**KZFSN2**	1.5608	36.41	23.33	10.0	1/10	1
3	**KZFSN2**	1.5608	36.41	23.33	10.0	1/10	1
4	**BK3**	1.5001	44.08	29.38	10.0	1/10	7
5	**KZFS1**	1.6164	29.51	18.25	14.5	1/7	2
6	**KZFS6**	1.5949	32.38	20.30	12.5	1/8	7
7	**KZFSN2**	1.5608	36.41	23.33	10.0	1/10	1
8	**KZFSN2**	1.5608	36.41	23.33	10.0	1/10	1
9	**KZFSN2**	1.5608	36.41	23.33	10.0	1/10	7
10	**KZFSN2**	1.5608	36.41	23.33	10.0	1/10	1
11	**KZFSN9**	1.6016	31.26	19.51	12.5	1/8	2
12	**LAK16A**	1.7369	34.67	19.96	16.0	1/6.25	2
13	**LAK28**	1.7478	33.96	19.43	16.0	1/6.25	2
14	**LAK31**	1.6997	37.94	22.32	14.5	1/5	1
15	**LAK33**	1.7574	35.12	19.98	16.0	1/7	2
16	**LAK8**	1.7162	36.12	21.05	20.0	1/5	2
17	**LAK9**	1.6940	36.72	21.67	14.5	1/7	1
18	**LAKL21**	1.6430	40.30	24.53	10.0	1/10	1
19	**LAKN14**	1.6998	37.23	21.90	14.5	1/7	1

Fig. 3 shows a graph representing the dependence between the aperture *D* and the main refractive index n_e for the glasses numbered as in tab. 5. It is obvious that high refractive indexes (>1.65) insure larger apertures.

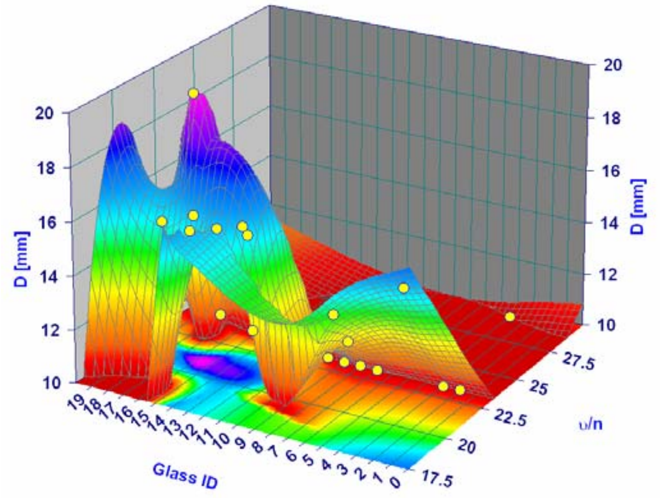


Fig. 4 Course of aperture in relationship with the parameter *k*

The dispersive properties are also significant. Theoretically, the Abbe number should be as small as possible. However, the Abbe number and the main refractive index are correlated. This is why the parameter k was introduced. For the glasses in the family short flint its optimum value is 18...19. For the glasses in the family crown – lanthanum best values of k are 19...21. These observations are illustrated in fig. 4.

4. Possibility to increase the aperture of apochromats by introducing aspherical surfaces

The paper suggests a method to improve the solutions with a small f-number or bring the low quality ones into the category of diffraction-limited as follows. E. g., Fig. 5 presents the solution FK54-LAK33-TIF6 which is diffraction limited (RMS OPD <0.07λ and Strehl ratio>0.80) up to the f-number f/6.25.

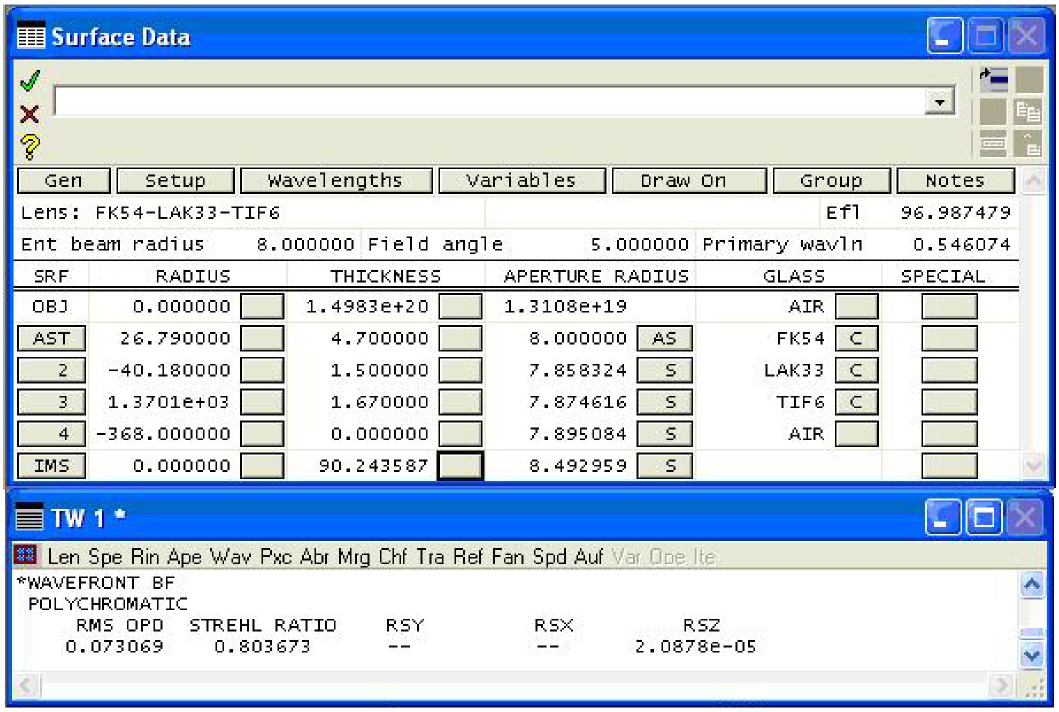


Fig. 5 Surface data and wavefront parameters at aperture f/6.25

A small increase of the aperture produces a fast loss of quality. E.g., at h=8 mm (f/6.25) the Strehl ratio is 0.803, as shown in fig. 5. At h=10 mm (f/5) the Strehl ratio becomes 0.209, which means a dramatic loss of image quality for the marginal zone. This large decrease is due to a fast increase of longitudinal spherical aberration, which is in a strongly nonlinear relationship to the height (the aberration is approximately proportional to the square of the height).

Indeed, as fig. 6 shows, the longitudinal spherical aberration has a very high gradient in respect with the height. The spherochromatism indicates that the apochromatism is preserved as well as the focal chromatic shift. That means the chromatic characteristics are independent of the incidence height and the vulnerable element is the spherical residual aberration.

Small changes of radii or thicknesses actually introduce only smaller or larger amounts of defocusing but have no effect on curves' shape; therefore bending is not efficient in changing the shape of spherochromatism curves.

The only possibility to change the spherochromatic curves' shape is to turn one of the spherical surfaces into an aspheric one. The paper suggests this transformation for the last surface (the fourth one).

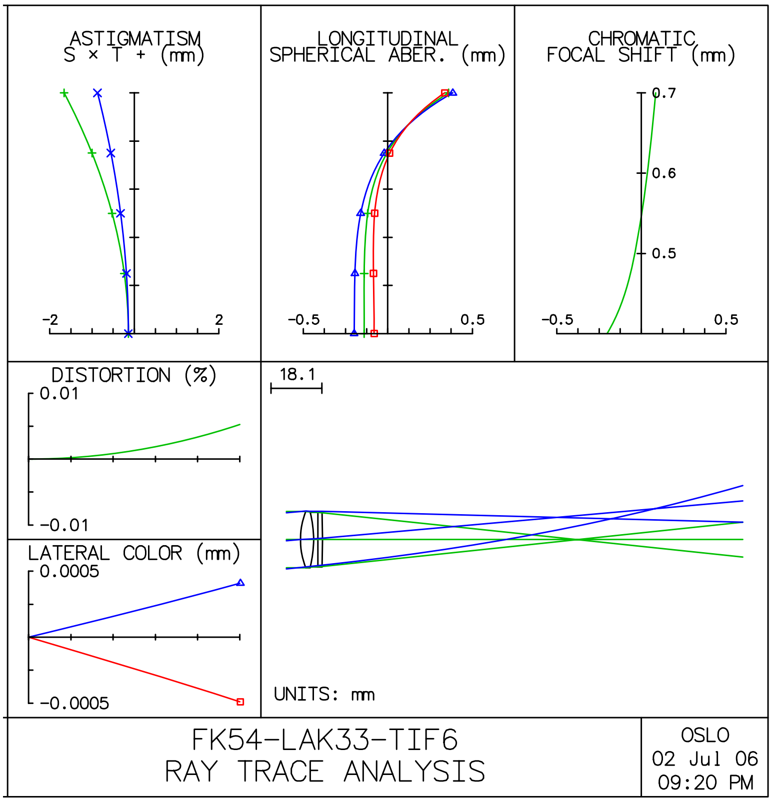


Fig. 6 Spherochromatism and chromatic focal shift at a double aperture (f/5)

As the thickness of the middle lens (TH2) controls the position of the spherochromatic curves along the optical axis and the conic constant of the forth surface (CC4) can modify the spherical residual aberration, the slider-wheel setup defined TH2 and CC4 as variable parameters.

A aspherical system, scaled to exactly f'=100mm is presented in fig. 7.

The conic constant, equal to 89, shows that the generating curve is an ellipse.

The conic constant and the radius of the osculate circle at the surface vertex are needed for the generating ellipse. These values are k=89 and r= 378.25 (which is the radius of the initial sphere).

Considering a and b the half-axis of the ellipse along the optical axis and, respectively, along the y-axis, the following relations are useful:

$$\begin{aligned} k &= \frac{b^2}{a^2} - 1 \\ r &= \frac{b^2}{a} \end{aligned} \quad . \tag{6}$$

Solving system (6) the solution is:

a=4.31 mm; b=40.43 mm.

The equation of the generating ellipse becomes:

$$z = \frac{r\left(1-\sqrt{1-(k+1)\frac{y^2}{r^2}}\right)}{k+1} = 28.586\left(1-\sqrt{1-5.56.10^{-4}y^2}\right). \tag{7}$$

5. Conclusions

The literature is poor in glass choice recommendations and the offer of optical glass is actually very rich. The designer must find some criteria based on reason in choosing suitab. sorts. Random choice is very inefficient and, in most cases, unsuccessful. The results of this paper offer a solution to this problem. The practical solutions designed by the authors provide very high image quality for apertures twice larger than the traditional ones.

A thorough analysis of refractive and dispersive properties of glasses explains the requirements of compatibility. Compatibility is regarded as a condition that insures a diffraction-limited solution for an apochromatic system. The authors insisted on the problem of residual spherochromatism, which limits the aperture and managed to provide solutions with much smaller f-numbers than the usual ones.

Original software was developed for glass compatibility and apochromat design. The results of running the programs consist in 88 triplets, of which 53 are diffraction–limited. The best of these are marked in tab. 3. The combinations of glasses in the same tab. may be regarded as practical recommendations in choosing compatible sorts for triplet design.

Aspherical surfaces can also bring substantial improvement in image quality for systems, which are mainly affected by spherical residual aberration. A proper generating noncircular curve is hard to find without specialized software and needs appropriate skills of the human operator.

References

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Садржај: *Пројектовање апхроматских система је отежано из два разлога: омпатибилности типа стакла и произвољног односа улаза c_1/c_a. Произвођачи стакла нуде висок распон врста тако да избор триплета компатибилних стакала постаје одвојен важан проблем. Овај рад даје решење за математичко моделирање компатибилности стакла и практично анализира врсте које нуди фирма Schott GmbH. Оригинални софтвер је нудио 22 компатибилна триплета. Аутори су разматрали могућности повећања односа c_1/c_a од вредности 0.6 која је дата у литератури до опсега од [0.5…0.8]. Значи они су пројектовали и анализирали сет од 88 триплета. Коректан избор стакла може осигурати двапут већу апертуру него традиционални за апохромате највећег квалитета (ограничени дифракцијом).*

Кључне речи: *Компатибилност оптичког стакла, дизајн триплета, велика апертура, асферична површина.*